IB MYP YEAR 5

**YEAR 10 Extended**
Mathematics

**Assessment #2**

# TRANSFORMATION OF FUNCTIONS

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Teacher: **Ms. Li, Mr. So & Mr. Wong**

Date of task: **Friday, November 2, 2012** Time allowed: **60 minutes**

| Student's **Performance** in Different **Criterion** | | | |
|---|---|---|---|
| **B** | 4 | **C** | 4 |

## INSTRUCTIONS:

- Read the instructions for all questions carefully.
- Show all work, steps and proper units.
- Ask the teacher for scrap paper, but any work on the scrap paper will **NOT** be marked.
- Write in **PENCIL**.
- **GDC** is allowed.
- Allowed to use **non-electronic dictionary**.

## ASSESSMENT:

- Read the criteria descriptors carefully before you start your work. This will give you a clear understanding of what is required and what a quality piece of work for this task must include. This way you give yourself the best chance of achieving the highest level in this task.
- This task assesses Criteria **B & C** considering ALL the questions.
  - ✧ Criterion **C** will be assessed as an **overall impression** on the presentation of work in this assessment.

Criterion B: INVESTIGATING PATTERNS

| Achievement level | Task Specific Rubric | IBO Published Descriptor | Student's self-evaluation |
|---|---|---|---|
| 0 | The student does not reach a standard described by any of the descriptors given below. | The student does not reach a standard described by any of the descriptors given below. | |
| 1–2<br>Do Maths | The student is able to<br>• Find **some** of the equations of the curves in **part A**.<br>• Recognize and describe **some** of the transformations in **part A**. | The student **applies, with some guidance**, mathematical problem-solving techniques to recognize **simple** patterns. | **(0-8)** |
| 3–4<br>General Rule | The student is able to<br>• Do **most** of the questions in **Part A** and **some** questions in **Part B**.<br>• Apply knowledge to unfamiliar situations in **Q10** and **Q11**. | The student<br>• **selects and applies** mathematical problem-solving techniques to recognize patterns, and<br>• **suggests** relationships or general rules. | **Teacher's Final Grade** |
| 5–6<br>Test it | The student is able to<br>• Fulfill the requirements above.<br>• Deduce a general form for **Q12**.<br>• Apply and justify the general from in **Q13**.<br>• Find another transformation that works in **Q14**. | The student<br>• **selects and applies** mathematical problem-solving techniques to recognize patterns,<br>• **describes** them as relationships or general rules, and<br>• **draws conclusions** consistent with findings. | **(0-8)** |
| 7–8<br>Prove it | The student is able to<br>• Fulfill the requirements above<br>• Prove mathematically the answer in **Q15**.<br>• Identify the unfamiliar transformation in **Q16** and find its equation in **Q17**. | The student<br>• **selects and applies** mathematical problem-solving techniques to recognize patterns,<br>• **describes** them as relationships or general rules,<br>• **draws conclusions** consistent with findings, and<br>• **provides justifications or proofs**. | |

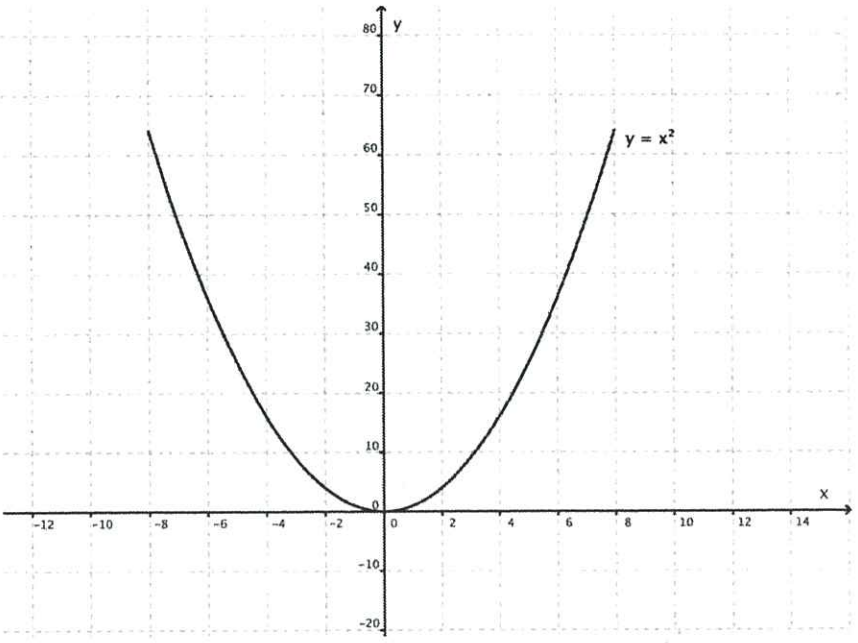
Criterion C: COMMUNICATION IN MATHEMATICS

| Achievement level | Task Specific Rubric | IBO Published Descriptor | Student's self-evaluation |
|---|---|---|---|
| 0 | The student does not reach a standard described by any of the descriptors given below. | The student does not reach a standard described by any of the descriptors given below. | (0-6) |
| 1–2 | There are **some errors** or inconsistencies in use of **terminology**.<br>There are **some** errors in the writing of **equations**.<br>Narrative is **difficult to follow**. | • The student shows **basic** use of mathematical language **and/or** forms of mathematical representation.<br>• The lines of reasoning are **difficult to follow**. | **Teacher's Final Grade** |
| 3–4 | Generally students use **correct terminology** accurately, with only a **few errors**.<br>Equations are **mostly** written **clearly** and accurately.<br>Narrative can be **followed**, and diagrams are **clear** and labeled. | • The student shows **sufficient** use of mathematical language **and** forms of mathematical representation.<br>• The lines of reasoning are **clear** though not always **logical** or **complete**.<br>• The student moves between different forms of representation **with some success**. | (0-6) |
| 5–6 | The student uses the correct terminology accurately for **most of the problems.**<br>Equations are **clear** and **accurate**<br>Narratives are **concise**, **logical** and **complete**.<br><br>**All** diagrams are **clear** and labeled | • The student shows **good** use of mathematical language **and** forms of mathematical representation.<br>• The lines of reasoning are **concise**, **logical** and **complete**.<br>• The student moves **effectively** between different forms of representation. | |

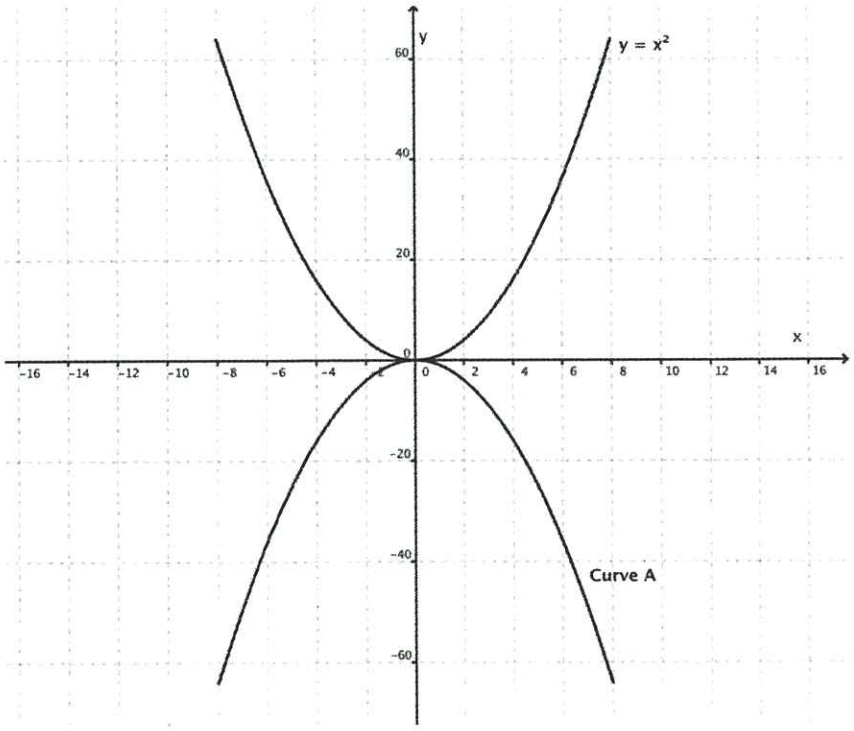
## PART A (Suggested time: 20 minutes)

When the designers of the Hong Kong MTR were creating the now-famous logo, they decided to use transformations of functions.

They started with a parabola as in the graph below:



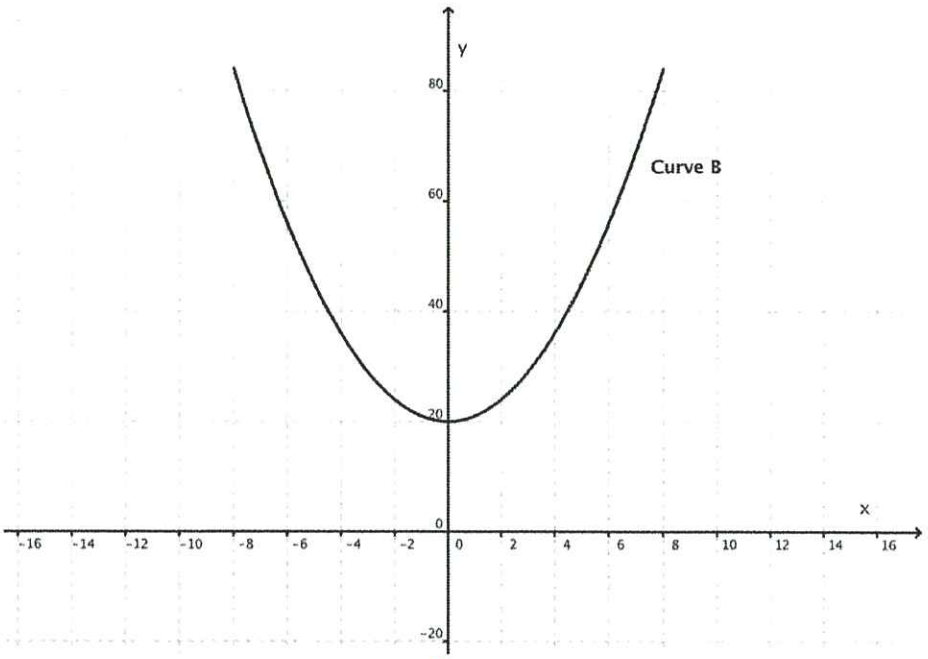

They then reflected this curve by the x-axis:




1. If the **starting curve** has the equation $y = x^2$, what is the **equation of curve A**?

$y = -x^2$

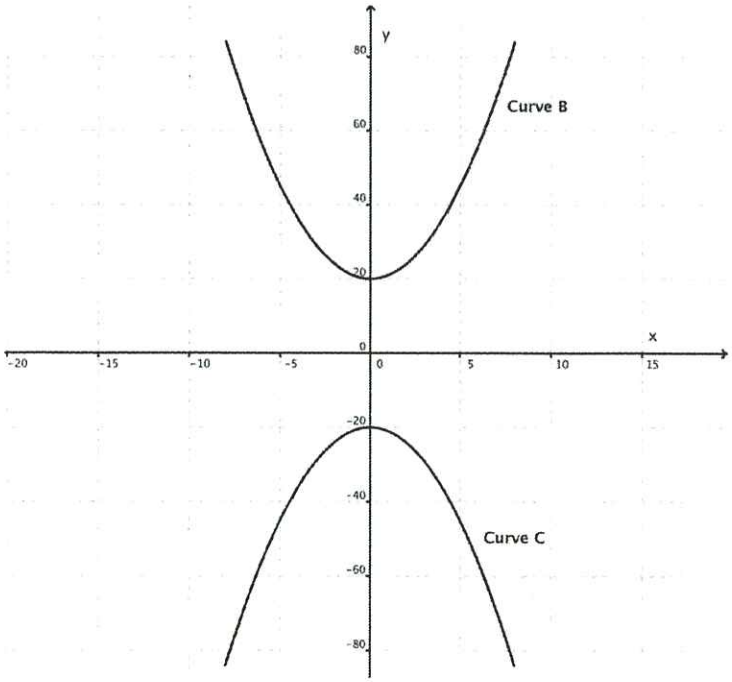
The designers then performed a transformation from their **starting curve**. It became this:




2. What do you think the **equation of curve B** is?

$y = x^2 + 20$ ✓

After the designers **reflected curve B** by the **x-axis**, they got **curve C**:



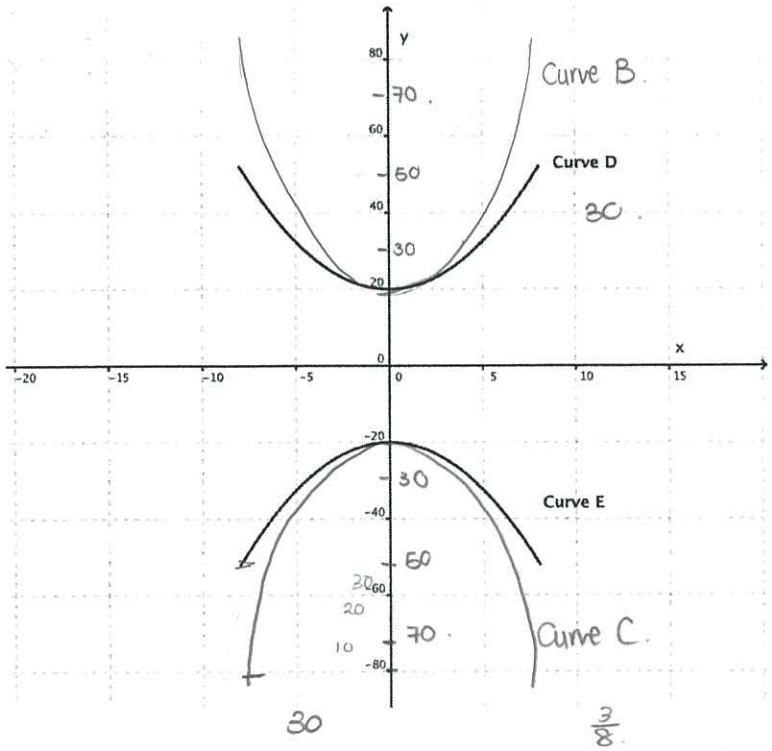

$y = f(-x)$ y-axis.

$y = -f(x)$ x-axis.

3. What do you think the **equation of curve C** is?

$y = -(x^2 + 20) = -x^2 - 20$ ✓

The designers then went on to make one further alteration. The diagram below shows the results of that alteration. (Curves D and E are sketched on the same axes as B and C were on the last graph):




4. Describe in words what this last alteration was (to make **curve B become curve D** and **C become E**)?

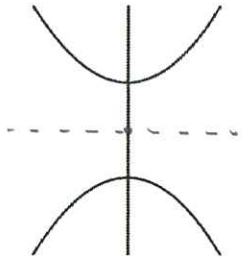
The last alternation from curve B to curve D was Contracting $\frac{3}{8}$ along y-axis and from curve C to curve E was contracting $\frac{3}{8}$ along y-axis.

5. What do you think the **equation of curve D** is?

$y = (\frac{3}{8}x)^2 + 20$

6. What do you think the **equation of curve E** is?

$y = -(\frac{3}{8}x)^2 - 20$

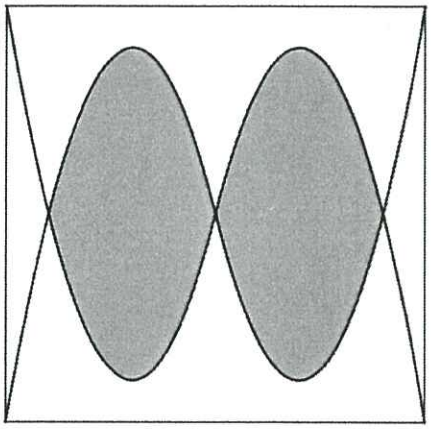


7. The "finished" logo is on the left. Describe fully the symmetry of the design

The symmetry of the "finished" logo on the left in showned with the dotted. The diagonal dotted seperates the logo into half, where each image is like the mirror image of the other image. After cutting with the dotted line, the image reflects each other, which make it a symmetry.

## PART B (Suggested time: 40 minutes)

Here is a logo of a well-known sportswear company:

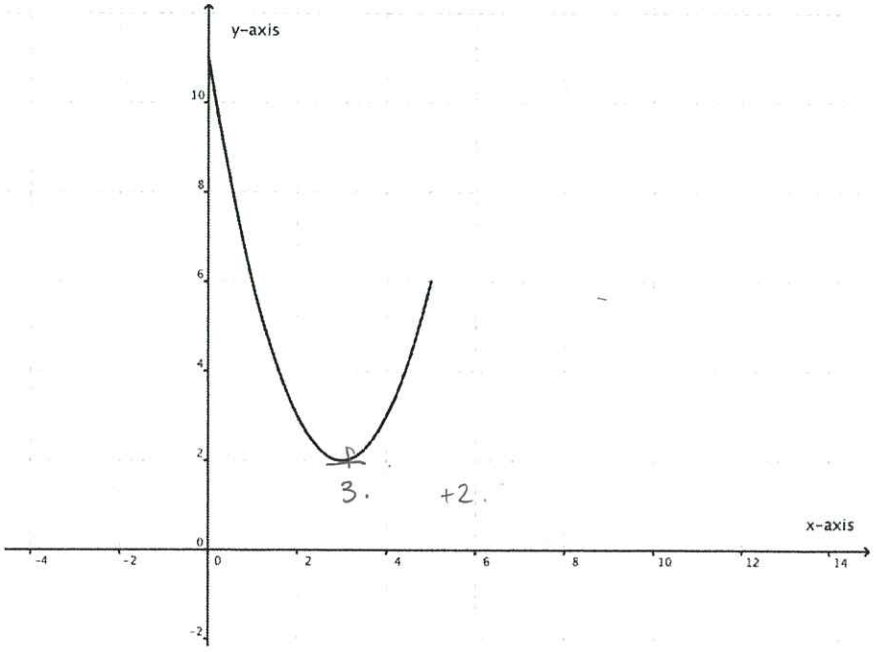


Two students, Ahmed and Delinda, are interested in the mathematics of the curves that make up the logo, and the relationship between the curves.

Ahmed believes that the design is really based on a simple curve (a quadratic), which is then transformed using simple transformations.

He says that the "basic unit" of the design below is

$$y = x^2 - 6x + 11 \qquad 0 \leq x \leq 5$$



8. Show that the equation $y = x^2 - 6x + 11$ can be written as the **second form** $y = (x - 3)^2 + 2$.

$y = (x^2 - 6x + 9) + 2$

$= [(x-3)(x-3)] + 2$

$= (x-3)^2 + 2$

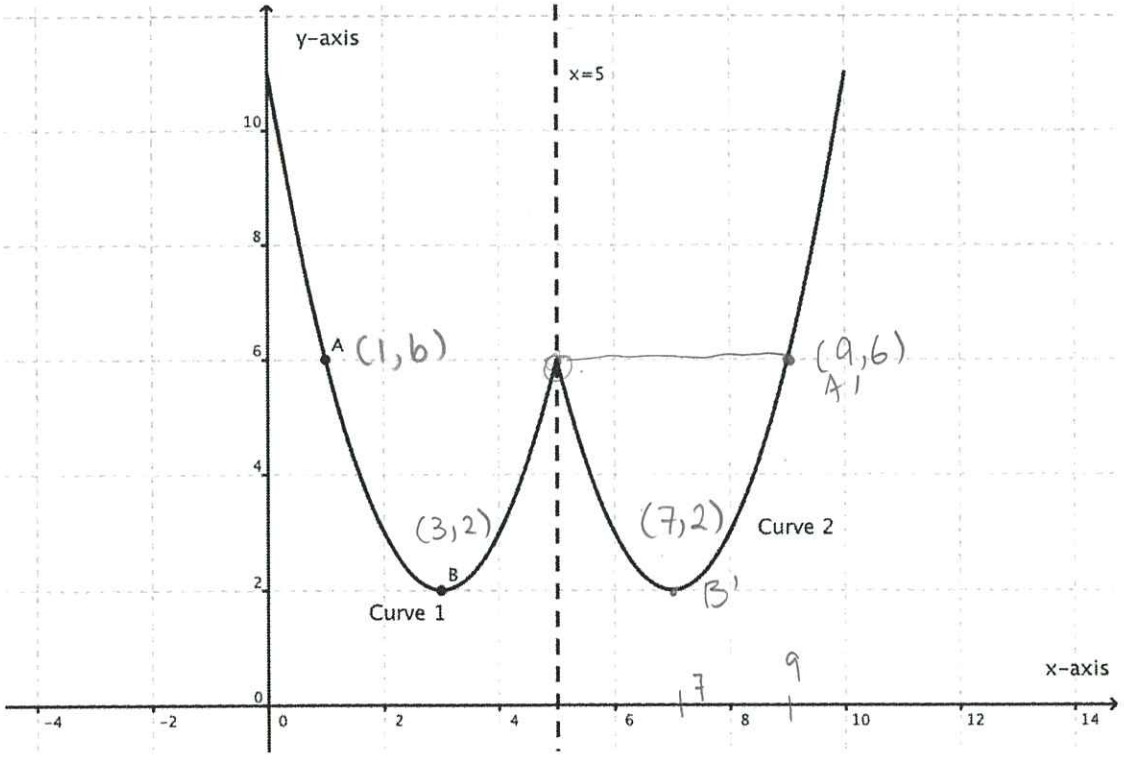
$x \quad -3$

$x \quad -3$

9. Describe how this second form of the equation matches up with key features of the graph?

The second form of the equation matches up with key features of the graph, which is the vertex (3,2). The vertex in the graph is exactly (3,2). This is taken from the equation $(x-h)^2 + k$, where $(h, k)$ is the vertex. So applying this equation to our equation $(x-3)^2 + 2$, we received that (3,2) is the vertex of the graph.

He goes on to say that this curve is **reflected in the line x = 5** as shown below:




10. A is the point (1, 6). What are the coordinates of the image of A after reflection in **x = 5**?

(9, 6)

11. B is the point (3, 2). What are the coordinates of the image of B after reflection in **x = 5**?

(7, 2)

12. P is the general point (x, y). What are the coordinates of the image of P after reflection in **x = 5**?

(5, 6)

13. Ahmed takes the x-coordinate of his answer to Q12, and substitutes this in for x in the original equation $y = x^2 - 6x + 11$. He (correctly) believes that it gives him the equation of curve 2. What equation for curve 2 does Ahmed get, and what range of values of x does it apply to?

$y = 5^2 - 6(5) + 11$ $= 25 - 30 + 11$ $= 6$,,

Curve 2 $\Rightarrow y = (x-5)^2 + 6$

$y = x^2 - 10x + 25 + 6$

$y = x^2 - 10x + 31$.

Only 5.

$0 < x < 5$

f.t.

Delinda believes that Curve 1 in the diagram above can be transformed into Curve 2 **by a different way**.

14. What transformation(s) is/are Delinda thinking of? Give as many details as possible.

Another transformation that Delinda is thinking of is reflect curve 1 along the x-axis, then translate 5 units to the right. By doing this transformation, Delinda receives exactly the same curve 2 as Ahmed did.

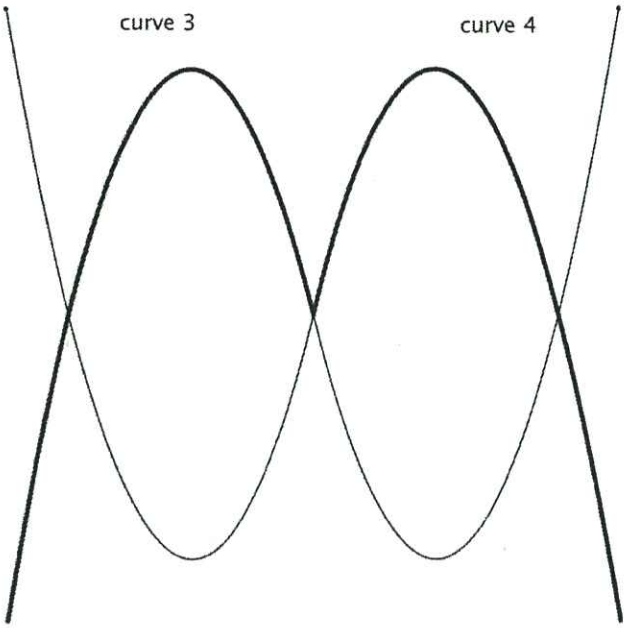
- curve 1
curve 2

From the first step, Delinda reflect the graph (curve 1) along x-axis, which creates ∩, with ↑∪→ and ↑J→ overlapping each other. Then move the point B in the curve two (exactly in curve 1) 5 units to the right, we will be ending up with a graph like this ↑W→, exactly like the one Ahmed created.

15. Is it possible for both Ahmed and Delinda to be right? Explain your answer.

It is possible that both Ahmed and Delinda to be right, since with their own theory, they each created a graph that looks exactly the same and even worked too. Ahmed directly reflected the graph in (x=5) to make the curve and Delinda took a longer way, which is reflecting the graph along x-axis and then move 5 units to the right. They ended up with the same curve, graph, so it is possible for both of them to be right.

f.t.




To complete the above logo, curve 1 is transformed into curve 3, and curve 2 is transformed into curve 4

16. What single transformation is performed on curves 1 and 2 in order to end up with the logo?

The transformation is reflecting along the $x$-axis.

17. What is the equation of curve 3?

Curve 1
$y = x^2 - 6x + 11$
or
$y = (x-3)^2 + 2$

Curve 3
$y = (-x)^2 - (6(-x)) + 11 = -x^2 + 6x + 11$
OR
$y = ((-x) - 3)^2 + 2 = (-x-3)^2 + 2$

**End of Assessment**